

Student Number

2022 Glenwood High School Year 12 - Trial HSC Examination

Mathematics Advanced

General Instructions

- Reading Time 10 minutes
- Working time 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet will be provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2-7)

- Attempt Questions 1- 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 40)

- Attempt Questions 11 34
- Allow about 2 hours and 45 minutes for this section

Section I

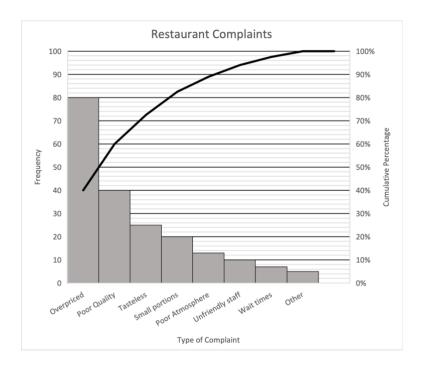
10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 10

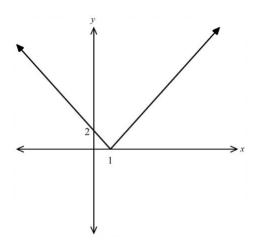
1. The Pareto chart below shows the frequency of types of complaints received by a restaurant.



What percentage of complaints were for poor quality?

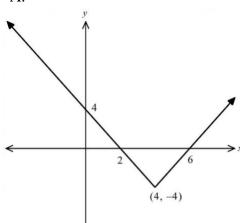
- A. 20%
- B. 40%
- C. 60%
- D. 80%

2. The graph of y = f(x) is shown below.

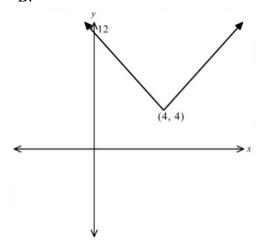


Which of the graphs below represents y = f(x + 3) + 4?

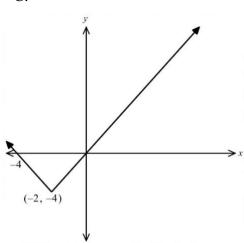
A.



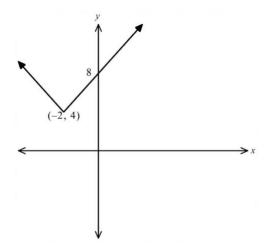
B.



C.



D.

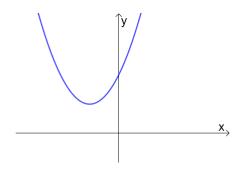


3. The stem and leaf plot shows the number of shots needed to complete a game of pool on two tables. Twenty games were played on each table.

Table A								Γ	able	В		
			7	6	4	0	6	8				
		8	4	4	1	1	2	6	7	7		
8	8	7	7	5	3	2	0	2	3	4	4	
		6	5	5	4	3	1	3	5			
				2	1	4	5	5	6	8	9	
						5						
					9	6	9					

Is the result of 69 shots an outlier for either of the tables?

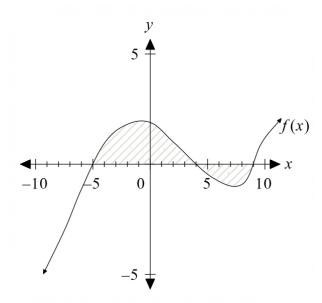
- A. No, it isn't for either table.
- B. Yes, for Table A only
- C. Yes, for Table B only
- D. Yes, for both tables
- **4.** The diagram shows the graph of $y = x^2 + bx + 2$



Which of the following could be the value of *b*?

- A. 2
- B. -2
- C. 3
- D. -3

5. The curve shown in the graph has x-intercepts at -5, 4, and 9. Which of the following integrals would NOT give the shaded area under the curve?



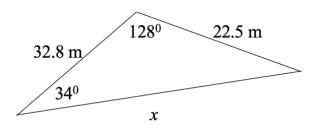
A.
$$A = \int_{-5}^{4} f(x) dx + \left| \int_{4}^{9} f(x) dx \right|$$

B.
$$A = \int_{-5}^{4} f(x) dx - \int_{4}^{9} f(x) dx$$

$$C. \quad A = \left| \int_{-5}^{9} f(x) \ dx \right|$$

D.
$$A = \int_{-5}^{4} f(x) dx + \int_{9}^{4} f(x) dx$$

6. The correct value for x is:



NOT TO SCALE

A.
$$x = 22.5 \times \sin 34^{\circ} \div \sin 128^{\circ}$$

B.
$$x = 22.5 \times \sin 128^{\circ} \div \sin 34^{\circ}$$

C.
$$x = \sqrt{22.5^2 + 32.8^2 - 2 \times 22.5 \times 32.8 \times \cos 18^\circ}$$

D.
$$x = \sqrt{22.5^2 + 32.8^2 - 2 \times 22.5 \times 32.8 \times \cos 34^\circ}$$

- 7. What is the gradient of the tangent to the function $y = \cos^2 x$ at $x = \frac{\pi}{6}$?
 - A. -1
 - B. $\frac{1}{2}$
 - C. $\sqrt{3}$
 - D. $-\frac{\sqrt{3}}{2}$
- **8.** Which of the following represents the domain of the function $f(x) = \frac{1}{\ln(x-2)}$?
 - A. $[2,\infty)$
 - B. $(2,\infty)$
 - C. $(2,3)\cup(3,\infty)$
 - D. $(3,\infty)$

9. It is known that f(x) is an odd function and g(x) is an even function.

Given that f(2) = 2 and g(2) = -2, what is the value of f(g(-2)) + g(f(-2))?

- A. -4
- B. -2
- C. 0
- D. 4
- 10. If $y = log_2 4x$ then $\frac{dy}{dx} =$
 - A. $\frac{\log_e 4x}{\log_e 8}$
 - B. $\frac{1}{x log_e 2}$
 - C. $\frac{\log_e 2x}{\log_e 2}$
 - D. $\frac{log_e 4}{log_e 2x}$

END OF SECTION I

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2022 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Student Number

Section II Answer Booklet 1

90 marks Attempt Questions 11 – 34 Allow about 2 hour and 45 minutes for this section

Booklet 1 — Attempt Questions 11–28 (62 marks)

Booklet 2 — Attempt Questions 29–34 (28 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 25-28 of Booklet 1. If you use this space, clearly indicate which question you are answering.

0	uestion	11	(2)	marks)	١

Exp	press $\frac{2x}{x^2-4}$	$-\frac{x+1}{x^2-x-2}$	as a single	algebraic fra	ction, in sim	plest form.		2
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		••••••					 	••
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••••		• • • • • • • • • • • • • • • • • • • •					 	
Que	estion 12 (5 marks)					 	
Fine	d the follow	ving deriv	atives.					
		U						
	$\frac{d}{dx}\Big(3x^5 +$							2
							 	2

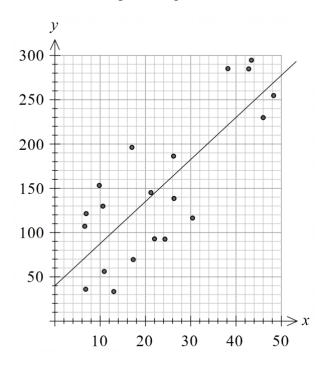
Question 12 continues on page 11

Question 12 (continued)

b)	$\frac{d}{dx}\left(\frac{e^x\cos(2x)}{x^2}\right)$	3
	estion 13 (2 marks)	_
Fin	d the following integral $\int \frac{6(x-4)}{x^2-8x} dx$	2

Question 14 (5 marks)

An environmental scientist is studying the results of bushland rehabilitation on native species. The scatterplot shows x, the number of native trees planted in various rehabilitated sites, and y, the number of native species present in that area. The least-squares regression line is shown on the scatterplot.



(a)	Use the least squares regression line to estimate the number of the native species present in an area with 30 native trees planted.							
(b)	What is the equation of the least-squares regression line?	2						

Question 14 continues on page 13

Question 14 (continued)

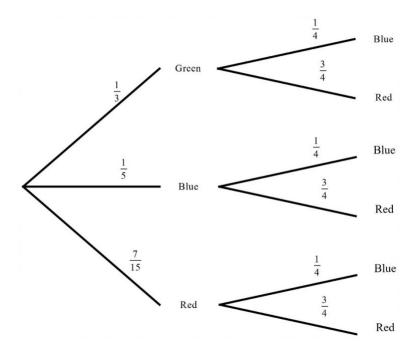
(c)	Hence or otherwise, predict the number of native trees that should be planted for approximately 410 native species to be present in the area.						
Qu	estion 15 (3 marks)						
The	e population of a seal colony varies according to the equation						
	$P = 100\sin\left(\frac{\pi t}{6}\right) + 300$						
whe	ere t is in months.						
(a)	Find the maximum and minimum populations of the colony.	1					
(b)	Find the number of months it takes for the population to vary from the maximum population to the minimum population.	2					

Question 16 (3 marks)

An artist has two boxes of paints beside his easel. The first box contains 15 tubes which contain green, red or blue paint and the second contains four tubes which have only red or blue paint.

The artist chooses one tube of paint from each box.

The tree diagram below shows the probabilities of him choosing a given colour.



(a)	What is the probability that the two tubes he chooses are red and blue?	1
(b)	What is the probability that the tubes he chooses are not the same colour?	2

Question 17 (4 marks)

The present value of an annuity of \$1 for various interest rates and periods is shown in the table.

Present Value of an Annuity of \$1

	Interest Rate per period							
Periods	0.25%	0.50%	0.75%	1.00%	2.00%	3.00%		
3	2.9851	2.9702	2.9556	2.9410	2.8839	2.8286		
6	5.9478	5.8964	5.8456	5.7955	5.6014	5.4172		
9	8.8885	8.7791	8.6716	8.5660	8.1622	7.7861		
12	11.8073	11.6189	11.4349	11.2551	10.5753	9.9540		
24	23.2660	22.5629	21.8891	21.2434	18.9139	16.9355		
36	34.3865	32.8710	31.4468	30.1075	25.4888	21.8323		
48	45.1787	42.5803	40.1848	37.9740	30.6731	25.2667		

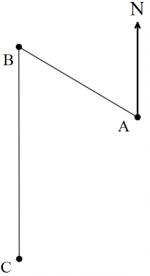
(a)	Find the amount, to the nearest cent, that must be invested quarterly at an interest rate of 3% p.a.
	compounded quarterly for 6 years, that would have a present value of \$20,000.
(b)	Find the future value of an annuity of $$1,000$ invested monthly at 0.5% interest compounded monthly for 1 year.

	Question	18	(2)	marks')
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	function $f(x)$	$x) = 4tan^3x \text{ is g}$	iven				2
		$2x(1+\tan^m x), \text{ fin}$		n.			_
	• • • • • • • • • • • • • • • • • • • •						
Qu	estion 19 (3	marks)					
The	e table below	shows the probab	bility distributio	n of a random va	riable <i>X</i> .		
	X	2	3	4	5		
	P(X)	0.4	0.1	0.27	0.23		
(a)	Calculate E	(X).				_	1
	•••••						
(b)	Calculate V	ar(X).					2

Question 20 (6 marks)

A plane flies 145 km from point A to point B on a bearing of 305°. The plane then flies 208 km due south to point C before returning to point A.

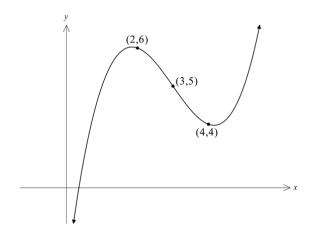


NOT TO SCALE

(a)	Complete the diagram with the information provided and find $\angle ABC$.	2
(b)	Find the distance the plane must travel from point C to return to point A, correct to the nearest kilometre.	2
(c)	What is the three-figure bearing from point C to point A?	2

Question 21 (2 marks)

The graph of y = f(x) is shown.



Use two applications of the trapezoidal rule to find an approximation for

r ²	1	
	f(x)	dχ

 	 •••••
 	 •••••
 	 •••••

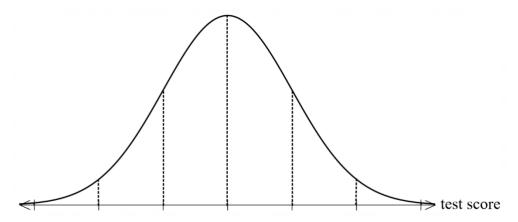
1

1

Question 22 (4 marks)

Marks on an aptitude test are normally distributed with a mean of 100 and a standard deviation of 15.

A normal distribution curve is shown below, where the vertical lines represent z-scores from -3 to 3.



- (a) For the normal distribution curve above, label the corresponding test marks (not z-scores) along the horizontal axis.
- (b) Shade under the normal distribution curve where (approximately) the top 20% of scores lie.
- (c) Hae-Won took the test and her z-score was 0.5. She needs to score in the top 20% in order to enrol in a training program.

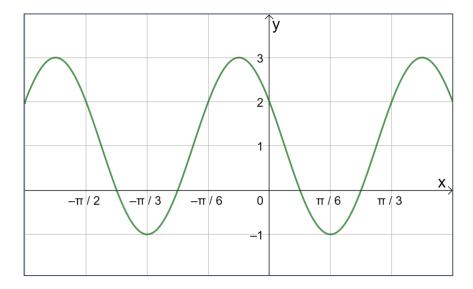
If she studies, retakes the test, and her test score increases by 10 marks, will she be able to enrol?

Justify your answer with calculations.

.....

Question 23 (5 marks)

The diagram shows the graph of $y = a\cos(bx+c)+d$.



(a)	State the range.	1
(b)	State the period.	1
(c)	Find the values of a , b , c and d .	3

3

3

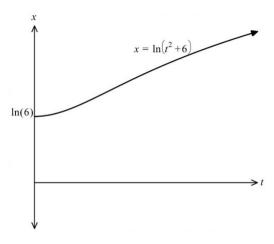
Question 24 (3 marks)

Describe the three transformations which, when applied in order, transforms the graph of $y = 2\sin x$ to the
graph with equation $y = \sin\left(\frac{x+\pi}{3}\right)$.
Question 25 (3 marks)
Prove that
$\frac{1+\tan^2 x}{1+\cot^2 x}=\tan^2 x$
Questions 11 – 25 are worth 52 marks in total

Question 26 (3 marks)

A particle moves on the x-axis so that it's displacement in metres from the origin at a time t second is given by the equation $x = \ln(t^2 + 6)$.

The particle starts from rest at the point $x = \ln(6)$ and accelerates in a positive direction as shown in the distance-time graph below.



Find the time when the acceleration of the particle is zero and the velocity at this time.	3

Question 27 (4 marks)

(a)	Show that $\frac{d}{dx}(3x^2e^{2x+1}) = 6xe^{2x+1}(x+1)$	2
(b)	Hence evaluate $\int_1^2 x(x+1)e^{2x+1} dx$.	2

Question 28 (3 marks)

A random variable Z is normally distributed with mean 0 and standard deviation 1.

3

The table gives the probability of this random variable being greater than different values of z.

Z	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
P(Z>z)	0.5	0.4013	0.3085	0.2266	0.1587	0.1057	0.668	0.0401	0.0228

In a hospital of 3125 patients, it is found that 708 patients have a body temperature higher than 36.8°C. The body temperatures are distributed normally with a mean μ and standard deviation 0.3°C.

Using the given table, find the value of μ , correct to two decimal places.

Proceed to Booklet 2 for Questions 29-34

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2022	TRIAL HIGH	HER SCHOOL	CERTIFICATE
	EXAMINAT	ION	

Student Number

Mathematics Advanced

Section II Answer Booklet 2

Booklet 2 — Attempt Questions 29–34 (28 marks)

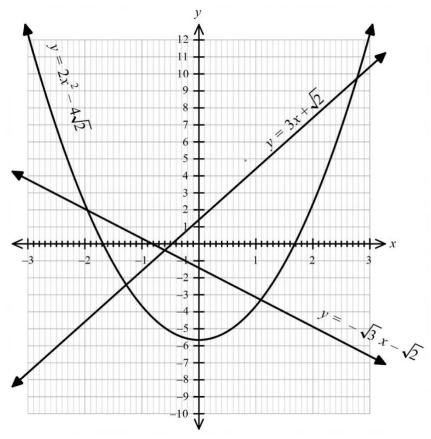
Instructions

- Write your Student Number at the top of this page.
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- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39-40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

2

Question 29 (4 marks)

The diagram below shows the graphs of $y = 3x + \sqrt{2}$, $y = -\sqrt{3}x - \sqrt{2}$ and $y = 2x^2 - 4\sqrt{2}$.



Use the graph to find the solution to the following inequalities, giving your answer correct to the nearest tenth.

(a) $3x + \sqrt{2} > 2x^2 - 4\sqrt{2}$

.....

(b)
$$2x^2 + \sqrt{3}x - 3\sqrt{2} \ge 0$$

.....

Question 30 (6 marks)

Find and classify the nature of any stationary points and points of inflection (horizontal or otherwise) for the function $y = 12x^5 - 15x^4 - 40x^3$.					
Hence, sketch the graph of the function, clearly labelling any stationary points, points of inflection and intercepts.					
Working space for Question 30 continues next page					

Question 31 (4 marks)

The volume of a pond decreases during a period of no rain.

The volume in the pond, V, after t days can be modelled by the equation

$$V = V_0 e^{-kt}$$

where V_0 is the initial volume of the pond.

(a)	Given that it takes 10 days for the volume of the pond to halve, show that $k \approx 0.0693$.	3
(b)	If there is 30 megalitres (ML) in the pond after 7 days, calculate the initial volume of the pond, to the nearest ML.	1

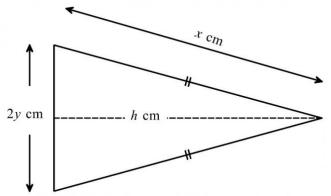
Question 32 (4 marks)

A continuous random variable X has a probability density function $f(x)$ given by						
	$f(x) = \begin{cases} Ax + \\ \end{cases}$	B, 0,	$1 \le x \le 6$ otherwise			
where A and B are constants.						
The median of X is 3. Find the value	ues of A and B.					
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		• • • • • • • •		• • • • • • • • • • • • • • • • • • • •		

2

Question 33 (6 marks)

A banner is designed as an isosceles triangle, with equal sides of length x cm and base of length 2y cm, as shown.



The total perimeter of the triangle is 40 cm.

(a)	Show that the area of the triangle in terms of x can written as:
	$A = (20 - x) (40x - 400)^{\frac{1}{2}}$

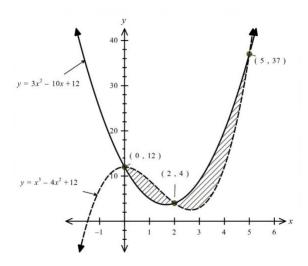
Question 33 continues on page 36

Question 33 (continued)

"	maximum area.						
	Hence, find this area.						

Question 34 (4 marks)

The curves $y = x^3 - 4x^2 + 12$ and $y = 3x^2 - 10x + 12$ intersect at the points (0, 12), (2, 4) and (5, 37) as shown on the graph below.



Calculate the total shaded area enclosed between the two curves.

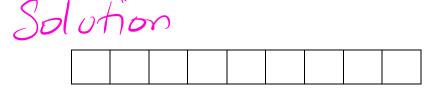
Working space for Question 34 continues next page

Question 34 (continued)
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Section II Extra writing space
If you use this space, clearly indicate which question you are answering.





Student Number

2022 Glenwood High School Year 12 Trial HSC Examination Assessment Task 4

Mathematics Advanced

General Instructions

- Reading Time 10 minutes
- Working time 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet will be provided
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Total marks: 100

Section I – 10 marks (pages 2 - 7)

- Attempt Questions 1- 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 40)

- Attempt Questions 11 34
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks Attempt Questions 1–10 Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

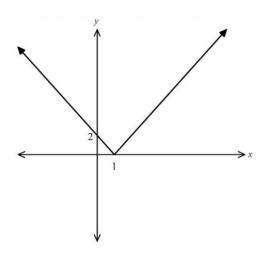
1. The pareto chart below shows the frequency of types of complaints received by a restaurant.



What percentage of complaints were for poor quality?

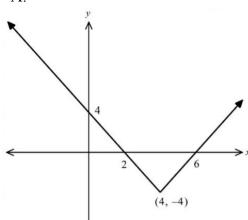
- (A.)
- 20%
- B. 40%
- C. 60%
- D. 80%

2. The graph of y = f(x) is shown below.

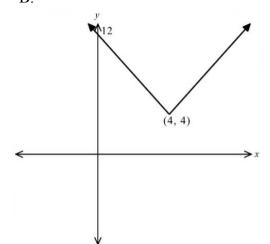


Which of the graphs below represents y = f(x + 3) + 4?

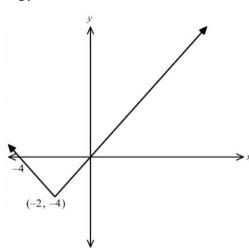
A.



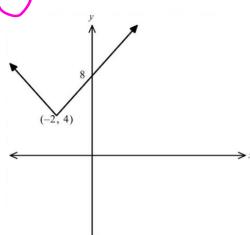
B.



C.



D.

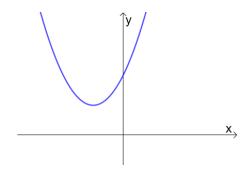


3. The stem and leaf plot shows the number of shots needed to complete a game of pool on two tables. Twenty games were played on each table.

	Τ	able	A						Γ	able	В	
			7	6	4	0	6	8				
		8	4	4	1	1	2	6	7	7		
8	8	7	7	5	3	2	0	2	3	4	4	
		6	5	5	4	3	1	3	5			
				2	1	4	5	5	6	8	9	
						5						
					9	6	9					

Is the result of 69 shots an outlier for either of the tables?

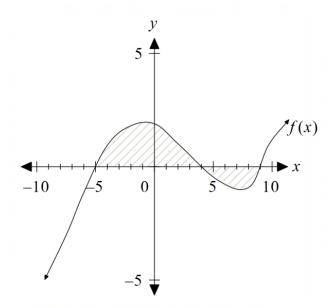
- A. No, it isn't for either table.
- B Yes, for Table A only
 - C. Yes, for Table B only
 - D. Yes, for both tables
- **4.** The diagram shows the graph of $y = x^2 + bx + 2$



Which of the following could be the value of *b*?

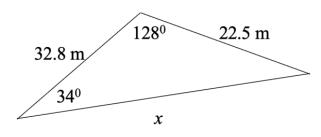
- A. 2
- B. -2
- C. 3
- D. -3

5. The curve shown in the graph has x-intercepts at -5, 4, and 9. Which of the following integrals would NOT give the shaded area under the curve?



- A. $A = \int_{-5}^{4} f(x) dx + \left| \int_{4}^{9} f(x) dx \right|$
- B. $A = \int_{-5}^{4} f(x) dx \int_{4}^{9} f(x) dx$
- - D. $A = \int_{-5}^{4} f(x) dx + \int_{9}^{4} f(x) dx$

6. The correct value for x is:



NOT TO SCALE

A.
$$x = 22.5 \times \sin 34^{\circ} \div \sin 128^{\circ}$$

$$\mathbf{B} \quad x = 22.5 \times \sin 128^{\circ} \div \sin 34^{\circ}$$

C.
$$x = \sqrt{22.5^2 + 32.8^2 - 2 \times 22.5 \times 32.8 \times \cos 18^\circ}$$

D.
$$x = \sqrt{22.5^2 + 32.8^2 - 2 \times 22.5 \times 32.8 \times \cos 34^\circ}$$

- 7. What is the gradient of the tangent to the function $y = \cos^2 x$ at $x = \frac{\pi}{6}$?
 - A. -1
 - B. $\frac{1}{2}$
 - C. $\sqrt{3}$
 - $\frac{\text{D}}{2} \frac{\sqrt{3}}{2}$
- **8.** Which of the following represents the domain of the function $f(x) = \frac{1}{\ln(x-2)}$?
 - A. $[2,\infty)$
 - B. $(2,\infty)$
 - $(2,3) \cup (3,\infty)$
 - D. $(3,\infty)$

9. It is known that f(x) is an odd function and g(x) is an even function.

Given that f(2) = 2 and g(2) = -2, what is the value of f(g(-2)) + g(f(-2))?

- (A.) -4
- B. -2
- C. 0
- D. 4
- 10. If $y = log_2 4x$ then $\frac{dy}{dx} =$
 - A. $\frac{\log_e 4x}{\log_e 8}$
 - $\frac{1}{x \log_e 2}$
 - C. $\frac{log_e 2x}{log_e 2}$
 - D. $\frac{log_e 4}{log_e 2x}$

END OF SECTION I

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Mathematics Advanced

Student Number

Section II Answer Booklet 1

90 marks Attempt Questions 11 – 34 Allow about 2 hour and 45 minutes for this section

Booklet 1 — Attempt Questions 11–28 (62 marks)

Booklet 2 — Attempt Questions 29–34 (28 marks)

Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 25-28 of Booklet 1. If you use this space, clearly indicate which question you are answering.

2

Question 11 (2 marks)

Express $\frac{2x}{x^2-4} - \frac{x+1}{x^2-x-2}$ as a single algebraic fraction, in simplest form. $= \frac{3x}{(x+1)(x-2)} - \frac{(x+1)}{(x-2)(x+1)}$ $= \frac{2x}{(x+2)(x-2)} - \frac{1}{(x-2)}$ $= \frac{2x - (x+2)}{(x+2)(x-2)}$ $= \frac{(x-2)}{(x+2)(x-1)}$

Question 12 (5 marks)

Find the following derivatives.

(a)
$$\frac{d}{dx} (3x^5 + \frac{2}{x^3} - 4\sqrt{x})$$

= $\frac{d}{dx} (3x^5 + 2x^{-3} - 4x^{1/2})$
= $15x^4 - 6x^{-4} - 2x^{-1/2}$
= $15x^4 - 6 - 2$
 $x^5 - \sqrt{x}$

Question 12 continues on page 11

Question 12 (continued)

(b)
$$\frac{d}{dx} \left(\frac{e^x \cos(2x)}{x^2} \right)$$

3

2

$$u = e^{x}$$

$$u' = e^{x}$$

$$dx \left(e^{x} \cos 2x\right)$$

$$dx \left(e^{x} \cos 2x\right)$$

$$= e^{x} \cos 2x - 2e^{x} \sin 2x$$

.....

$$u' = e^{x} \cos 2x$$

$$u' = e^{x} \cos 2x - 2e^{x} \sin 2x$$

$$v' = 2x$$

$$\frac{d}{dx}\left(\frac{e^{x}\cos(2x)}{x^{2}}\right) = \left(\frac{e^{x}\cos(2x) - 2e^{x}\sin(2x)}{x^{2}}\right)^{2} - 2xe^{x}\cos(2x)$$

Question 13 (2 marks)

Find the following integral

 $\int \frac{6(x-4)}{x^2 - 8x} dx$

$$= \int \frac{6x - 24}{x^2 - 8x} dx$$

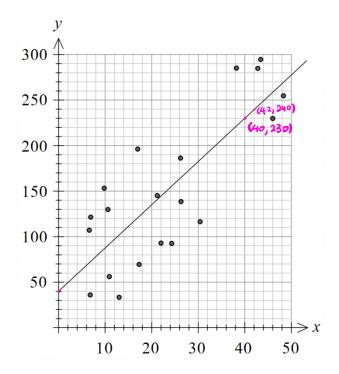
$$= \int \frac{3(2x - 8)}{x^2 - 8x} dx$$

= $3 \ln(2^2 - 8x) + C$

2

Question 14 (5 marks)

An environmental scientist is studying the results of bushland rehabilitation on native species. The scatterplot shows x, the number of native trees planted in various rehabilitated sites, and y, the number of native species present in that area. The least-squares regression line is shown on the scatterplot.



(a) Use the least squares regression line to estimate the number of the native species present in an area with 30 native trees planted.

180

(b) What is the equation of the least-squares regression line?

 $y = \frac{19}{4} \times +40 \qquad \Rightarrow 5$

y=5x+40

.....

Question 14 continues on page 13

1

Question 14 (continued)

(c) Hence or otherwise, predict the number of native trees that should be planted for approximately 410 native species to be present in the area.

 $y = \frac{19}{4} \times + 40$ or y = 5x + 40

 $410 = \frac{19}{2}x + 40 \qquad 5x = 370$

4 24 = 74

 $\frac{19}{4}x = 370$

2 × 78

Question 15 (2 marks)

The population of a seal colony varies according to the equation

$$P = 100 \sin\left(\frac{\pi t}{6}\right) + 300$$

where *t* is in months.

(a) Find the maximum and minimum populations of the colony.

 $\max = 400 \quad \max \text{ occurs } \omega \text{ ben } Sin \theta = 1$

 $Min = 200 \qquad \theta = \frac{\pi}{4} \Rightarrow \frac{\pi t}{6} = \frac{\pi}{4} \Rightarrow t = 3$

min occurs when $\sin \theta = -1$ $\theta = \frac{37}{5} \Rightarrow \frac{17}{5} = \frac{37}{5} \Rightarrow t = 9$

(b) Find the number of months it takes for the population to vary from the maximum population to the minimum population.

 $P = \frac{2\pi}{4} = 12$ from a) t = 9-3

6 months between maximum and

.....

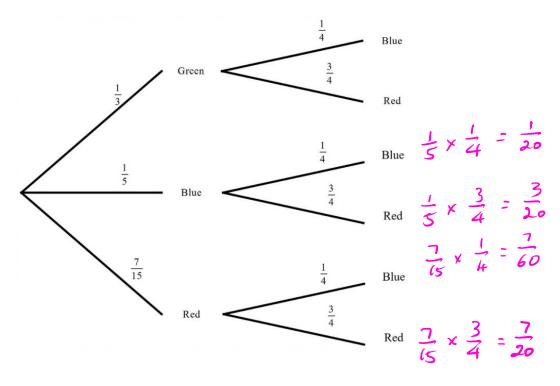
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Question 16 (3 marks)

An artist has two boxes of paints beside his easel. The first box contains 15 tubes which contain green, red or blue paint and the second contains four tubes which have only red or blue paint.

The artist chooses one tube of paint from each box.

The tree diagram below shows the probabilities of him choosing a given colour.



(a) What is the probability that the tubes he chooses are red and blue?

P(R and B) = P(RB) + P(BR) $= \frac{7}{60} + \frac{3}{20}$ $= \frac{16}{60} = \frac{4}{60}$

(b) What is the probability that the tubes he chooses are not the same colour?

P(Same) = P(RR) + P(BB) $= \frac{7}{20} + \frac{1}{20}$ $= \frac{8}{20}$

P(not same) = 1 - P(same) $= 1 - \frac{8}{20}$

Question 17 (4 marks)

The present value of an annuity of \$1 for various interest rates and periods is shown in the table.

Present Value of an Annuity of \$1

		r reserre va	146 01 411711	marcy or φ_{\perp}	-										
	Interest Rate per period														
Periods	0.25%	0.50%	0.75%	1.00%	2.00%	3.00%									
3	2.9851	2.9702	2.9556	2.9410	2.8839	2.8286									
6	5.9478	5.8964	5.8456	5.7955	5.6014	5.4172									
9	8.8885	8.7791	8.6716	8.5660	8.1622	7.7861									
12	11.8073	11.6189	11.4349	11.2551	10.5753	9.9540									
24	23.2660 22.5629 21.8891 21.2434 18.9139 16.9355														
36	34.3865 32.8710 31.4468 30.1075 25.4888 21.8323														
48	45.1787	42.5803	40.1848	37.9740	30.6731	25.2667									

(a)	Find the amount, to the neares	st cent, that must be invested	quarterly at an interest rate of	of 3% p.a.
	compounded quarterly for 6 y	rears that would have a prese	ent value of \$20,000.	

$$\Gamma = 0.75\%$$
 $\Omega = 24$

$$A = $913.70$$

Question 18 (2 marks)

The function $f(x) = 4tan^3x$ is given.

2

If $f'(x) = 12 \tan^2 x (1 + \tan^m x)$, find the value of m.

$$f(x) = 4 (\tan x)^3$$

$$f'(x) = 12 \tan^2 x \sec^2 x$$

.....

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.....

Question 19 (3 marks)

The table below shows the probability distribution of a random variable X.

X	2	3	4	5
P(X)	0.4	0.1	0.27	0.23

(a) Calculate E(X).

E(x) = 2x0.4 + 3x0.1 + 4x0.27 + 5x0.23

= 3.33

(b) Calculate Var(X).

2

1

 $Var(x) = E(x^2) - [E(x)]^2$

$$= 2^{2} \times 6.4 + 3^{2} \times 6.1 + 4^{2} \times 0.27 + 5^{2} \times 0.23 - (3.33)^{6}$$

= 1.48

.....

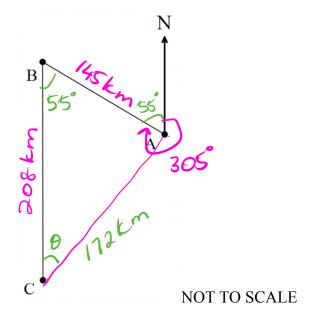
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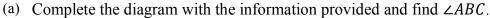
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Question 20 (6 marks)

A plane flies 145 km from point A to point B on a bearing of 305°.

The plane then flies 208 km due south to point C before returning to point A.





LNAB = 360° - 305° = 55°

LABC = 55° (alternate angles)

(b) Find the distance the plane must travel to return from point C to point A, correct to the nearest

kilometre.

Ac = \(\frac{145^2 + 208^2 - 2\times 145\times 208 \times \cos 55^2}{\tag{6}}

= 172.310...

(c) What is the three-figure bearing from point C to point A?

<u> Sin 0 - Sin 55</u>

145 176

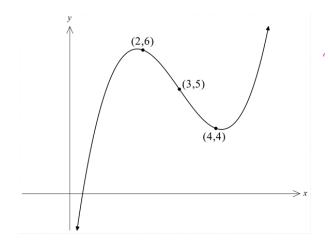
Sin 0 = (45 Sin 55

8 <u>43-6</u>

Bearing is 044°7

Question 21 (3 marks)

The graph of y = f(x) is shown.



Use two applications of the trapezoidal rule to find an approximation for

$$\int_{2}^{4} f(x) \, dx$$

 Z	2	3	14	h=	1	
 f(x)	6	5	4	_		
	·					
 Hx) dz		• • • • • • • • • • • • • • • • • • • •		• • • • • • • • • • • • • • • • • • • •	
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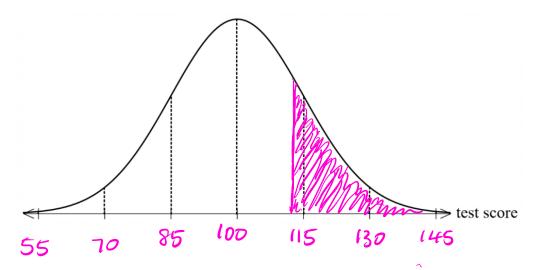
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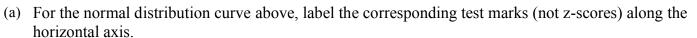
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Question 22 (4 marks)

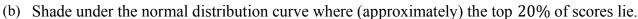
Marks on an aptitude test are normally distributed with a mean of 100 and a standard deviation of 15.

A normal distribution curve is shown below, where the vertical lines represent z-scores from -3 to 3.





ann Mydragram m



MM Magran Juli

(c) Hae-Won took the test and her z-score was 0.5. She needs to score in the top 20% in order to enrol in a training program.

If she studies, retakes the test, and her test score increases by 10 marks, will she be able to enrol? Justify your answer with calculations.

For
$$z = 0.5$$
, $0.5 = x - 100$

x = 107.5

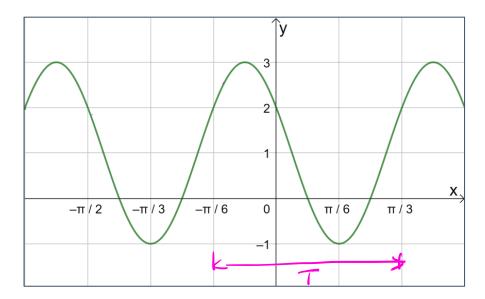
Z= 117.5 -100

1

3

Question 23 (5 marks)

The diagram shows the graph of $y = a\cos(bx+c)+d$.



(a) State the range.

[-1,3<u>]</u>

.....

(b) State the period.

State the period. $T = \frac{\pi}{4} + \frac{\pi}{3}$

(c) Find the values of a, b, c and d.

 $a=2 \qquad d=1$

 $\frac{2\pi}{6} = \frac{\pi}{2}$, b = 4

u = 0 · · · (/· · · · ·) · · ·

 $y = 2\cos(4x + c) + 1$ (0,2) is on the

2 = 2 ws c +1

 $2\cos c = 1$

 $\cos c = 1 \qquad c = T$

Question 24 (3 marks)

Describe the three transformations which, when applied in order, transforms the graph of $y = 2\sin x$ to the graph with equation $y = \sin\left(\frac{x+\pi}{3}\right)$.

28 in x to 8 in x Vertical dilation 8 f 1/2

Sin x to Sin (x) Horizontal dulation Sf 3

Sin $\left(\frac{x}{3}\right)$ to Sin $\left(\frac{x+y}{3}\right)$ Horizontal translation π units left.

Question 25 (3 marks)

Prove that $\frac{\sin^2 x + \cot^2 x - \csc^2 x - \cos^2 x}{1 + \cot^2 x} = \frac{1 + \tan^2 x}{1 + \cot^2 x}$

 $LH8 = \frac{1 + \tan 2\ell}{1 + \cot^2 \kappa}$

= Secx cosec x

= 1 x Sin x

= 8in X

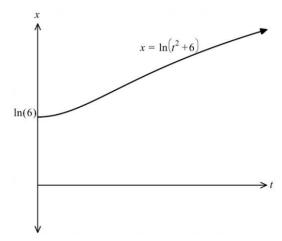
= tan x = RHS

Questions 11 – 25 are worth 52 marks in total

Question 26 (3 marks)

A particle moves on the x-axis so that it's displacement in metres from the origin at a time t second is given by the equation $x = \ln(t^2 + 6)$.

The particle starts from rest at the point $x = \ln(6)$ and accelerates in a positive direction as shown in the distance-time graph below.



Find the time when the acceleration of the particle becomes zero and find the velocity at this time.

 $x = \ln(t^2 + 6) \qquad v = \dot{z} = \frac{2t}{t^2 + 6}$

 $A = \frac{1}{x} = 2(t^2 + 6) - 4t^2$ u = 2t u' = 2 u' = 2

 $v = t^{2} + 6$ $= \frac{(2 - 2t)^{2}}{(4 - 2t)^{2}}$ v' = 2t

 $\ddot{x} = 0$ when $12 - 2t^2 = 0$

 $2(6-t^2)=0$

t = 56 (positre value only)

 $a + \tau = 16$

V = 2.16 $(16)^2 + 6$

= <u>256</u> 12

 $v = \frac{\sqrt{6}}{6} m/s$

Question 27 (4 marks)

(a) Show that $\frac{d}{dx}(3x^2e^{2x+1}) = 6xe^{2x+1}(x+1)$

2

 $\frac{d}{dx}(3x^2e^{2x+1})$ u' = 6x $v = e^{2x+1}$

 $\frac{d\chi}{d\chi} = \frac{2\chi + 1}{\sqrt{2\chi + 1}}$ $\frac{2\chi + 1}{\sqrt{2\chi + 1}} = \frac{2\chi + 1}{\sqrt{2\chi + 1}}$

 $= 6xe^{2x+1} + 6x^2e^{2x+1}$

as required

.....

(b) Hence evaluate $\int_1^2 x(x+1)e^{2x+1} dx$.

2

 $\int x(x+1) e^{2x+1} dx = \int 3x^2 e^{2x+1} \int 2x^2 e^{2x+1}$

 $= \frac{1}{6} \left[(3(4)e^{5}) - (3(1)e^{3}) \right]$

 $=\frac{1}{6}\left[12e^{5}-3e^{3}\right]$

 $= 2e^5 - \frac{1}{2}e^3$

Question 28 (3 marks)

A random variable Z is normally distributed with mean 0 and standard deviation 1.

3

The table gives the probability of this random variable being greater than different values of z.

Z	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
P(Z>z)	0.5	0.4013	0.3085	0.2266	0.1587	0.1057	0.668	0.0401	0.0228

In a hospital of 3125 patients, it is found that 708 patients have a body temperature higher than 36.8°C. The body temperatures are distributed normally with a mean μ and standard deviation 0.3°C.

Using the given table, find the value of μ , correct to two decimal places.

Let T be body temperature

PCT > 36.8) = 708 \(\times 0.2266

From the table, P(Z > 0.75) = 0.2266

Z=0.75 relates to temperature 36.8°C

 $Z = \chi - \mu$

 $0.75 = 36.8 - \mu$

 $36.8 - \mu = 0.225$

μ=36.58°C

.....

......

Proceed to Booklet 2 for Questions 29-34

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Section II Answer Booklet 2

Booklet 2 — Attempt Questions 29–34 (28 marks)

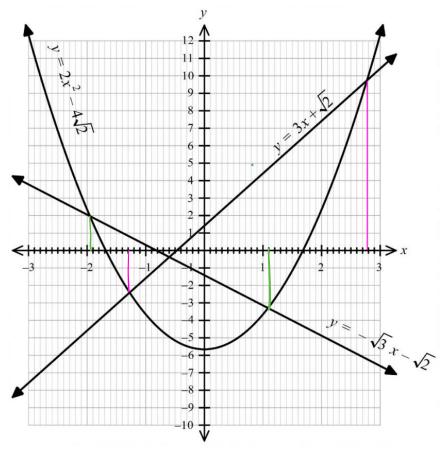
Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39-40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

2

Question 29 (4 marks)

The diagram below shows the graphs of $y = 3x + \sqrt{2}$, $y = -\sqrt{3}x - \sqrt{2}$ and $y = 2x^2 - 4\sqrt{2}$.



Use the graph to find the solution to the following inequalities, giving your answer correct to the nearest tenth.

(a)
$$3x + \sqrt{2} > 2x^2 - 4\sqrt{2}$$

From the graph, -1.3 < 2 < 2.8

.....

(b)
$$2x^2 + \sqrt{3}x - 3\sqrt{2} \ge 0$$

 $2\chi^2 - 3\sqrt{2} \ge -\sqrt{3}\chi$ (Subfract $\sqrt{3}\chi$ both sides)

 $2\pi^2 - 4\sqrt{2} \ge -\sqrt{3}\chi - \sqrt{2}$ (Subtract $\sqrt{2}$ both 8ides)

or Find where $y = 2x^2 - 4\sqrt{2}$ is above $y = -\sqrt{3}x - \sqrt{2}$

 $\alpha = -\frac{13 \pm \sqrt{(3)^2 - 4(2)(-3/2)}}{4}$ $\alpha \leq -1.95$ and $\alpha \geq 1.1$



Question 30 (6 marks)

Use calculus to determine and classifying the nature of stationary points and points of inflection (horizontal or otherwise) for the function $y = 12x^5 - 15x^4 - 40x^3$.

6

Hence, sketch the graph of the function, clearly labelling all key features. Accurate values for x-intercepts are **not** required.

 $y = 12x^5 - 15x^4 - 40x^3$

 $y' = 60x^4 - 60x^3 - 120x^2$

 $=60x^{2}(x^{2}-x-2)$

 $y' = 60x^{2}(x-1)(x+1)$

Stationary points, y'=0

 $\chi = 0$, $\chi = 2$, $\chi = -1$

 $y'' = 240x^3 - 180x^2 - 240x$

 $=60x(4x^2-3x-4)$

at x = 0, y" = 0 May be horizontal point of inflict.

y=0 at (0,0)

Test concavity change at x = -0.5, y'' = 45

at x = 0.5, y'' = -135

i. (CO,0) is a horizontal point of inflection

at x = 2, y'' = 720 > 0 Minimum turning

g = -110 u = -120 < 0 (heave a transfer of call -116)

y = 13, y = -180 D maximum Torning paint at (-1, 13)

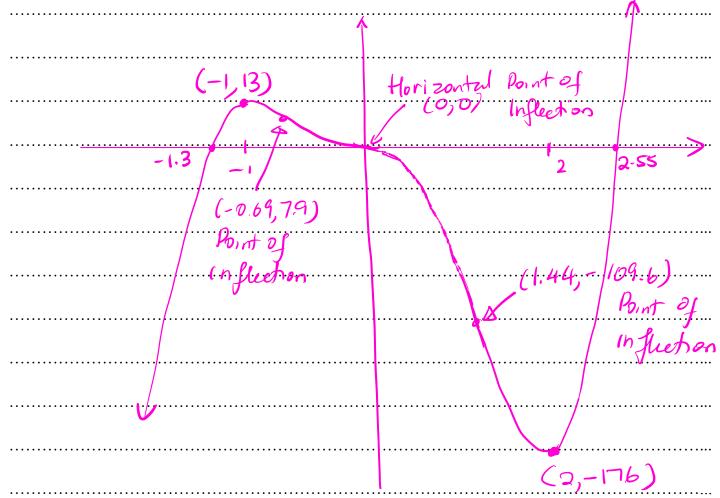
Points of influetion, y"=0

x = 0, $x = \frac{52715}{8} \approx -0.69$, 1.44

Working space for Question 30 continues next page

Question 30 (continued)

Cheek	Conce	vity cha	inges at		etan p	oints		
æ	-1	1-0.69	0 -0.5	./ /	1.44	2		
y"	-180	0	45	-180	0	720		
9	13	7.9			-109.6			
Points of influetion at (-0.69, 7.9) and								
(1.44, -109.6)								



 $\chi = \frac{15 \pm \sqrt{2145}}{24}$ $\chi = \frac{15 \pm \sqrt{2145}}{24}$ $\chi = 2.55, -1.3$

Question 31 (4 marks)

The volume of a pond decreases during a period of no rain.

The volume in the pond, V, after t days can be modelled by the equation

$$V = V_0 e^{-kt}$$

where V_0 is the initial volume of the pond.

(a)	Given that it takes	10 days for the volu	ume of the pond to ha	alve, show that $k \approx 0.0693$.
-----	---------------------	----------------------	-----------------------	--------------------------------------

 $V=0.5V_0$, t=10

0.5 V0 = V0 e

2 = 0.5

-10k = ln 0.5

k = <u>ln 0.5</u>

= 0.0693147...

k≈ 0.0693

.....

(b) If there is 30 megalitres (ML) in the pond after 7 days, calculate the initial volume of the pond, to the nearest ML.

at t=7, V=30ML

30 = Vo e -7k

k = ln 0.5

 $V_o = 30$ -76

= 48.735 ≈ 49ML

Question 32 (4 marks)

A continuous random variable X has a probability density function f given by

 $f(x) = \begin{cases} Ax + B, & 1 \le x \le 6 \\ 0, & otherwise \end{cases}$

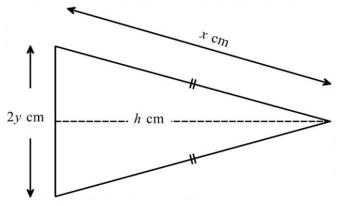
where A and B are constants.

The median of X is 3. Find the values of A and B.

18A + 6B - A - B = 1

Question 33 (6 marks)

A banner is designed as an isosceles triangle, with equal sides of length x cm and base of length 2y cm, as shown.



The total perimeter of the triangle is 40 cm.

(a) Show that the area of the triangle in terms of x can written as:

 $A = (20 - x) (40x - 400)^{\frac{1}{2}}$ 2x + 2y = 40

2y = 40-2x

y=20-x

Pythagoras' $y^2 + h^2 = 2^2$ $h^2 - x^2 + y^2$

 $h = \sqrt{x^2 - y^2}$

 $=\sqrt{\chi^2-(20-\chi)^2}$

 $h = \sqrt{40x - 400} = (40x - 400)^2$

Area = 5 × 2y × h

 $= (20 - x) (40x - 400)^{2} \text{ as required}$

Question 33 continues on page 36

Question 33 (continued)

(b) Use calculus to find the values of x and y which give a maximum area, showing why this gives the **4** maximum area.

Find this area. $\frac{dA}{dx} = -(40x - 400)^{1/2} + \frac{1}{2}(20 - x)(40x - 400)^{1/2}(40) \quad u = 20 - x$ $= -(40x - 400)^{1/2} + 20(20 - x) \quad u' = -1$ $(40x - 400)^{1/2} \quad v = (40x - 400)^{1/2}$ $= \frac{20(20 - x) - (40x - 400)}{(40x - 400)^{1/2}} \quad v' = \frac{-1/2}{(40x - 400)^{1/2}} \quad v' = \frac{-1/2}{(40x - 400)^{1/2}} \quad v' = \frac{1}{2}(40x - 400)^{1/2} \quad v' = \frac{1}{2}(40x - 400)^{1/2}$

 $= \frac{400 - 20x - 40x + 400}{(400 + 400)^{2}}$

 $\frac{dA}{dx} = \frac{800 - 60x}{(40x - 400)^{1/2}} \qquad A'' = -60 - (800 - 60x) 20$ $\frac{dA}{dx} = \frac{800 - 60x}{(40x - 400)^{1/2}} \qquad \sqrt{40x - 400} \qquad (401 - 400)$

For maximum area, $\frac{dA}{dx} = 0$ at $7c = \frac{40}{3}$

 $800-60x=0 \qquad A'=-5.2 < 0$ $2=\frac{40}{3}=13\frac{1}{3} \qquad \text{here maximum one}$

Test using 1st derivative to show $x = \frac{40}{3}$ gives maximum. When x = 13 $\frac{d4}{dx} \approx 1.82$

 $x = 14 \frac{dA}{dx} = -3.16$

... Max area at $x = \frac{40}{3}$ and y = 20 - x = 20 - 40

 $y = \frac{20}{3}$

Maximum area

 $A = (20 - \frac{40}{3}) \left(40 \left(\frac{40}{3}\right) - 400\right)^{\frac{1}{2}} = 400\overline{\sqrt{3}}$ $A = 76.98 \text{ cm}^{2}$

Question 34 (4 marks)

The curves $y = x^3 - 4x^2 + 12$ and $y = 3x^2 - 10x + 12$ intersect at the points (0, 12), (2, 4) and (5, 37) as shown on the graph below.

 $y = 3x^{2} - 10x + 12$ $y = 3x^{2} - 10x + 12$ $y = x^{3} - 4x^{2} + 12$ -1 1 20 (2, 4) (2, 4) 1 1 2 3 4 5 6

Calculate the total shaded area enclosed between the two curves.

Area = $\int_{-\infty}^{\infty} x^3 - 4x^2 + 12 - (3x^2 - 10x + 12) dx +$

 $\int_{3}^{3} x^{2} - 10x + 12 - (x^{3} - 4x^{2} + 12) dx$

 $= \int_{0}^{2} x^{3} - 7x^{2} + 10x \, dx + \int_{2}^{3} x^{3} + 7x^{2} - 10x \, dx$

 $= \left[\frac{2^{9}}{2} - \frac{72^{3}}{2} + 52^{2}\right]^{2} + \left[-2^{9} + \frac{72^{3}}{2} - 52^{2}\right]^{5}$

 $= \left[\frac{2}{4} - \frac{1x}{3} + 5x^{2} \right]_{0} + \left[\frac{-x}{4} + \frac{7x}{3} - 5x^{2} \right]_{2}$

 $= \left[\left(\frac{2^4}{4} - \frac{56}{3} + 20 \right) - 0 \right] + \left[\left(\frac{(5^4)}{4} + \frac{875}{3} - 125 \right) - \left(\frac{-2^4}{4} + \frac{56}{3} - 20 \right) \right]$

 $= \frac{16}{3} + \frac{125}{12} + \frac{16}{3}$ $= \frac{16}{3} + \frac{63}{12}$

Area = 253 units2

= 21/12 ande

Working space for Question 34 continues next page

END OF EXAMINATION